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THE DISTRIBUTION OF HARM IN PRICE-FIXING CASES

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The distribution of harm in price-fixing cases*

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Abstract

We consider a vertically related industry and analyze how the total harm due to a price increase upstream is distributed over downstream firms and final consumers. For this purpose, we develop a general model without making specific assumptions regarding demand, costs, or the mode of competition. We consider both the case of homogeneous and differentiated goods markets. Furthermore, we discuss data requirements and suggest explicit formulas and regression specifications that can be used to estimate the relevant terms in the harm distribution in practice, even if elevated upstream prices are rather constant over time. The latter can be achieved by considering perturbations of the demand curve. This in turn can be used to construct a supply curve for the case of imperfect competition that includes perfect competition and monopoly as special cases. Finally, we illustrate how basic intuition from the tax incidence literature carries over to the distribution of harm.

Keywords: cartel, abuse of a dominant position, pass on defence, apportionment of harm, supply curve, tax incidence.

JEL classification: D43; L42; L13

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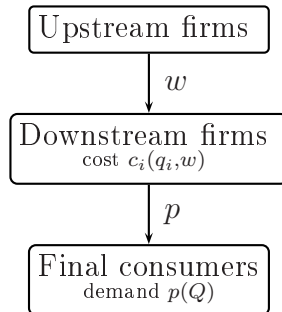


Figure 1: The simple vertical industry structure considered in this paper

1. Introduction

In this paper we consider a simple vertical industry structure as shown in Figure 1. There is an upstream sector with firms producing an input for the downstream sector which uses the input to produce a final good that is sold to consumers. We assume that due to cartelization or the abuse of a dominant position, the upstream sector is able to raise the wholesale price of the intermediate good. This will most likely have a negative effect on direct purchasers as the elevated wholesale price leads to a cost increase for direct purchasers. However, the direct purchasers might be able to pass on some or all of the harm they suffer to final consumers by increasing their price. The question we want to answer is how the total harm due to the increased upstream price is distributed over downstream firms and final consumers.

This analysis is motivated by recent, and perhaps more importantly, likely future developments of the legal framework of antitrust policy with respect to the issues of *pass-on defence* and the *legal standing of indirect purchasers* or class actions for consumers. In the setup considered in this paper in which an upstream firm illegally raised the wholesale price, pass-on defence refers to the possibility that the upstream firm (defendant) can have a downstream firm's (plaintiff) claim reduced by the amount that the latter passed on to consumers by means of a higher consumer price. Legal standing of indirect purchasers concerns the question whether or not indirect purchasers (in the context of our paper: consumers) who do not directly deal with the law infringer are allowed to bring an action before a court. We will review the development of the relevant antitrust law and policy in the US and in the EU in some detail in section 2 below. What comes out of this review is that in both of these jurisdictions, (some form of) pass-on defence and legal standing of indirect purchasers is in place or is very likely to be established in the near future. The establishment of these two pieces of legislation can be predicted to lead to an increase of court cases in which the correct distribution (or "apportionment") of antitrust harm down the production or supply chain needs to be determined.

However, so far there is a lack of a general framework that comprises the full range of competitive models (from perfect competition to monopoly) and incorporates several modes of competition (e.g. price or quantity competition in a homogeneous or heterogeneous market) in which this apportionment can be analyzed. With this paper we hope to contribute towards

filling this gap.¹ In the following we outline the contents of this paper.

Surely, an antitrust damage case would at some point start with the determination of the total harm.² However, this is a task that we do not concern ourselves with in this paper. The reason being that this has been done in various other papers.³ Hence, we assume that the total harm is given and exclusively concentrate on the distribution of harm in a simple vertical structure as shown in Figure 1.⁴

We do not model the upstream sector and simply assume that due to cartelization or the abuse of a dominant position the wholesale price, w , has been inflated.⁵

Taking the total harm as given, we determine the distribution of this total harm in proportion to actual losses suffered in the downstream sector and, due to pass on, on the level of final consumers. For this purpose, we first determine the change of downstream industry profits and consumer welfare in response to an increase in w , and then consider the share of the total actual harm (loss in downstream industry profits plus loss in consumer welfare) borne by consumers. We refer to this share as the consumer harm share (CHS).

In section 3 we start our analysis with a general homogeneous good model without making specific assumptions regarding demand, costs, or the mode of competition. We show that the CHS is decreasing in the downstream industry price-cost margin (PCM) and the price elasticity of demand and increasing in the Herfindahl-Hirschmann index (HHI) of downstream industry concentration and the downstream output elasticity with respect to the input price. The CHS turns out to be independent of the number of downstream firms affected. Clearly, if some downstream firms source from outside the upstream cartel or if they are vertically integrated with upstream firms (and therefore not affected by the cartel) this affects the *total* harm due to the cartel. Further, there is a distribution effect between downstream firms (where the unaffected firms gain and the others lose from the upstream cartel). However, this does not affect the *distribution* of harm between downstream firms and consumers.

¹We review the relevant previous literature on this issue below.

²As in Basso and Ross (2007), we will distinguish between “harm” which refers to losses in economic surplus of downstream producers and consumers and “damages” which refers to the legal term used to denote payments to be made by defendants. For instance, in the U.S. firms can sue for damages which are three times the harm inflicted.

³Earlier studies determining the harm are Baker and Rubinfeld (1999), Basmann and Boisso (1999), Connor (2001), Connor (2007), Finkelstein and Levenbach (1983), Fisher (1980), Harrison (1980), Page (1996), Rubinfeld (1985), Rubinfeld and Steiner (1984), and White (2001). More recent contributions are Brander and Ross (2006), Kosicki and Cahill (2006), Hellwig (2006), Verboven and Dijk (2007), and Basso and Ross (2007).

⁴Higher in the production chain, there could be more layers between upstream firms and final consumers. For instance, one can think of manufacturing firms, selling to wholesalers, wholesalers selling to retailers who then sell to final consumers. To keep the exposition simple, we focus on the case of upstream firms, downstream firms and final consumers.

⁵We will not explicitly deal with the relationship between the absolute illegal gain of the upstream sector and the absolute loss of direct and indirect purchasers. We just illustrate this issue with the following example. Assume final consumer demand is $P = 1 - Q$ and assume that there are m (n) Cournot firms upstream (downstream). One can then show that the illegal gain of upstream firms from raising the wholesale price above the subgame perfect equilibrium level w^* to $(1 + \delta)w^*$ (where $\delta > 0$) is larger than the sum of downstream industry profits and consumer surplus, as long as $\delta < 2m(2n + 3)/(n + 2)$. See also Schinkel, Tuinstra and Rüggeberg (2005) and Basso and Ross (2007) on this issue.

In section 3.1 we consider an extension of the basic model to allow for differentiated goods. It turns out that in this case the CHS hardly changes compared to the basic model.

We illustrate our results with various examples assuming specific forms of demand and production costs. These examples show that whenever it is possible and appropriate to make specific parametric assumptions regarding demand and costs, the expression for the CHS can become very simple. For instance, assuming linear demand for a homogeneous good and (asymmetric) constant marginal production costs, the CHS only depends on the number of firms and the conjectural variations parameter and is independent of demand parameters, marginal production costs, and the wholesale price.

Clearly, the usefulness of the framework put forward in this paper hinges on whether it can be applied in actual antitrust cases at reasonable costs. Hence, in section 4 we suggest feasible procedures to estimate the relevant terms in the CHS. Our suggestions here come in two parts. First, as suggested by e.g. Harrington (forthcoming) cartels have the tendency to keep the wholesale price w fairly constant over time. This might make it problematic to actually estimate the effect of an elevated wholesale price w that enters the CHS via downstream firms' cost functions. To circumvent this problem, we show in section 4.1 that instead of using shifts in w one can exploit ("equivalent") shifts in demand to estimate the CHS. This can be done by considering certain perturbations of the demand curve (brought about by demand shifters). A nice implication of this procedure is that it allows us to construct a supply curve for the case of imperfect competition that includes perfect competition and monopoly as special cases. With the help of the supply curve we can, in turn, illustrate that basic intuition from the tax incidence literature carries over to the distribution of harm in a vertically related industry. To be more precise, the incidence of a per-unit tax in e.g. a competitive market is determined by the slopes of the demand and supply curve. We show that this insight carries over to the context of our paper where the role of a tax is played by the wholesale price w . Second, in section 4.3 we elaborate on how the various building blocks of the CHS (elasticities and market indicators like PCM and HHI) can be estimated in practice. For this purpose we discuss data requirements and suggest explicit formulas and regression specifications that can be used to estimate the building blocks of the CHS. Moreover we discuss several potential problems of the estimation process such as endogeneity issues.

Finally, section 5 concludes. The appendix contains the proofs of the results.

Related literature: We are not the first trying to answer the question how total harm is distributed over a production or supply chain. First, our analysis is related to the incidence of an excise tax. An overview of this literature is given in Fullerton and Metcalf (2002). Second, there is also an extensive literature on the pass-through rate of price increases in (vertical) industry structures. See for instance Kosicki and Cahill (2006) and the references therein. Note that instead of concentrating on pass through rates of prices, we determine the distribution of harm with respect to lost profits or lost consumer welfare. Third, there is a recent literature that deals with the correct determination of damages in a vertically related industry. The common starting point of these papers is criticism of the so-called overcharge as a measure of harm in price-fixing cases. The overcharge is the difference between the anticompetitively elevated price and the price under competitive circumstances multiplied by the number of units purchased

at the elevated price. Hellwig (2006) determines the change in profits of a downstream firm affected by an illegally raised input price. In particular, he decomposes the overall change of profits into three different effects (a per-unit revenue effect, a business-loss effect, and a cost effect). Verboven and Dijk (2007) suggest a general framework to determine *discounts* on the overcharge as a measure of harm to downstream firms in price-fixing cases. As in Hellwig (2006), Verboven and Dijk (2007) also show that the overall change in downstream firms' profits can be decomposed into three effects (direct cost effect, pass-on effect, and output effect). Basso and Ross (2007) determine the total harm to downstream firms and final consumers when the price of a downstream input is raised upstream. They also provide measures of the distribution of harm between direct and indirect purchasers. However, in their analysis they rely on specific parametrizations of demand and costs. Finally, Han, Schinkel and Tuinstra (2008) consider a vertical industry structure with an arbitrary number of layers and assess the accuracy of the use of the overcharge as a correct measure of antitrust damages. Moreover, they assess damages of suppliers of a cartel in case the latter is in operation further down the supply or production chain.

Our paper differs in at least two main respects from these papers. First, we do not make assumptions on the mode of competition between downstream firms. Firms may for example compete in prices, quantities or price cost margins. Second, unlike the papers discussed above we devote considerable space to the practical issues concerning the actual estimation of our measure of the distribution of harm.

2. Pass-on defence and indirect-purchaser standing in the US and in the EU

In this section we review the evolution of antitrust law regarding pass-on defence and legal standing of indirect purchasers both in the US and the EU. Note that below we do not argue in favor or against a legal system that allows pass-on defence or legal standing of indirect consumers. We just wish to establish that in the current (and in likely future legal systems) there is room for pass-on defence and legal standing of indirect purchasers such that an analysis as the one we carry out in this paper might be useful and welcome.

Regarding the development in the U.S., the starting point is the 1968 Supreme Court decision in *Hanover Shoe, Inc. v United Shoe Machinery Corp.*⁶ in which it was ruled that the defendant could not use a pass on defence to avoid liability. Roughly, the reasoning behind this ruling was that the task of showing the extent of pass on “would normally prove insurmountable.” An additional reason was that indirect purchasers might be too dispersed and their claims likely to be small such that they “would have only a tiny stake in a lawsuit and little interest in attempting a class action.” In this case, “those who violate the antitrust laws by price fixing or monopolizing would retain the fruits of their illegality because no one was available who would bring suit against them.”

In 1977, in *Illinois Brick Co. v Illinois*⁷ the Supreme Court ruled that only direct but not

⁶Hanover Shoe, Inc. v. United Shoe Machinery Corp., 392 U.S. 481 (1968).

⁷Illinois Brick Co. v. Illinois, 431 U.S. 720 at 730-31.

indirect purchasers would be allowed to sue for antitrust harms. This can be viewed as a logical implication of the earlier ruling in the *Hanover Shoe* case: if a pass on defence is not allowed there is no room for indirect purchaser claims. In other words, if indirect purchasers were given legal standing, the extent of pass on would have to be determined which would contradict the earlier ruling in *Hanover Shoe*.

With these two rulings in place (no pass on defence and no standing for indirect purchasers) our analysis sketched above would hardly be necessary or relevant. But these two rulings constitute various problems. First, the *Hanover Shoe* ruling opened the doors for direct purchasers to claim the entire overcharge that occurred even if they passed on some or all of this overcharge to their customers.⁸ This would imply unjustified windfall profits for direct purchasers. Second, the *Illinois Brick* case implies that there is no compensation for other parties that suffered damages (e.g. indirect purchasers or final consumers). Accordingly, the two rulings have been criticized from the beginning and in response things have changed. In 1989 the Supreme Court ruled in *California v ARC America Corp*⁹ that indirect purchasers may sue for trebled damages under state law although damages suffered by direct purchasers may have been assessed by federal law. Kosicki and Cahill (2006) report that currently 23 states and the District of Columbia have so-called *Illinois Brick* repealer statutes that give indirect purchasers standing under state law. Finally, the Antitrust Modernization Committee (2007), henceforth AMC, rigorously assessing the U.S. antitrust law, gives the following advice to Congress:

“Direct and indirect purchaser litigation would be more efficient and more fair if it took place in one federal court for all purposes, including trial, and did not result in duplicative recoveries, denial of recoveries to persons who suffered injury, and windfall recoveries to persons who did not suffer injury. To facilitate this, Congress should enact a comprehensive statute with the following elements: Overrule *Illinois Brick* and *Hanover Shoe* to the extent necessary to allow both direct and indirect purchasers to sue to recover for actual damages from violations of federal antitrust law. [...] Damages should be apportioned among all purchaser plaintiffs—both direct and indirect—in full satisfaction of their claims in accordance with the evidence as to the extent of the actual damages they suffered.” (AMC, p.267).¹⁰

All these developments and facts (together with consumer class actions which are common in the U.S.) suggest that efficient methods are needed to determine how damages due to unlawful price increases are distributed (or apportioned) over the production chain.

With regard to the EU, it seems fair to say that the (case) law is at a less advanced state especially with respect to the passing on defence in antitrust cases. The annex to the Commission’s Green Paper on “Damages actions for breach of the EC antitrust rules” summarizes the situation regarding the issue of a passing on defence as follows: “It can be said that there is no passing on defence in Community law; rather, there is an unjust enrichment defence [...]” (Commission (2005), Annex p.48), henceforth Annex. This assessment seems to have emerged

⁸This is what is called “unjustified enrichment” in European Court rulings. More on this below.

⁹*California v. ARC America Corp.*, 490 U.S. 93 (1989).

¹⁰http://www.amc.gov/report_recommendation/amc_final_report.pdf

from relatively recent court cases in which firms claimed compensation for illegal duties and levies imposed by individual member states. Indeed, in *Comateb*¹¹ the European Court of Justice (ECJ) states: “Accordingly, a Member State may resist repayment to the trader of a charge levied in breach of Community law only where it is established that the charge has been borne in its entirety by someone other than the trader and that reimbursement of the latter would constitute unjust enrichment.” Furthermore, the ECJ’s states in its ruling in *Courage*¹²: “[T]he Court has held that Community law does not prevent national courts from taking steps to ensure that the protection of the rights guaranteed by Community law does not entail the unjust enrichment of those who enjoy them [...]”¹³ This statement is considered by some observers as a positive stand towards a pass-on defence. Others contradict this interpretation (see Norberg (2005), p.16ff).

But also in the EU a pass-on defence is met with considerable scepticism as the view that necessary computations are potentially very difficult. In fact, the Commission states that “It does not appear possible to construct a model which accurately identifies, at reasonable cost, the harm suffered by players at different levels of the supply chain.” (Annex, p.46). Nevertheless, the Commission also acknowledges that: “The door to apportionment is opened by the Court’s recognition of partial passing on in *Comateb* and *Michailidis*¹⁴.” Surely, it is one of the purposes of this paper to show that such an analysis can be accomplished and to show how the apportionment works.

With regard to the legal standing of indirect purchasers the situation in the EU seems to be clearer. In the *Courage* case, the ECJ states in §26: “The full effectiveness of Article 85 [now 81] of the Treaty and, in particular, the practical effect of the prohibition laid down in Article 85(1) [now 81(1)] would be put at risk if it were not open to any individual to claim damages for loss caused to him by a contract or by conduct liable to restrict or distort competition.” (See also the *Manfredi* case.¹⁵) This statement is interpreted by most observers to say that both direct and indirect purchasers can claim damages.

In any case, with the recent publication of the White Paper on “Damages actions for breach of the EC antitrust rules”, the Commission emphasizes that damage actions are a high priority in the EU. In fact, in its White Paper the Commission clearly argues in favor of allowing pass-on defence and legal standing of indirect purchasers. With respect to the first issue, the Commission states “defendants should be entitled to invoke the passing-on defence against a claim for compensation of the overcharge.” (White Paper, p.8) and with respect to the latter “In the context of legal standing to bring an action, the Commission welcomes the confirmation by the Court of Justice that **“any individual”** who has suffered harm caused by an antitrust infringement must be allowed to claim damages before national courts. This principle also applies to **indirect purchasers**, i.e. purchasers who had no direct dealings with the infringer, but who nonetheless may have suffered considerable harm because an illegal overcharge was

¹¹C-192/95 *Comateb and others v Directeur général des douanes et droits indirects* [1997] ECR I-165.

¹²C-453/99 *Courage Ltd v. Bernhard Crehan and Bernhard Crehan v. Courage Ltd.*, [2001] E.C.R. I-6297.

¹³Note also that Waelbroeck and Even-Shoshan (2004), p.6, state that “passing on defence was considered possible in Denmark, Germany (by some courts) and Italy where the question had arisen.”

¹⁴C-442/98 *Michailidis* [2000] ECR I-7145.

¹⁵Joined Cases C-295-298/04, *Manfredi*, [2006] ECR I-6619.

passed on to them along the distribution chain.” (White Paper, p. 4, original emphasis). Furthermore, the White Paper also suggests policy measures regarding collective redress of “scattered and relatively low-value damage” of individual consumers and small businesses that would allow the “aggregation of the individual claims of victims of antitrust infringements.” (for details see White Paper, p.4)

Taken together, the development in Europe also hints at the importance of developing methods to determine not only the exact amount of damage caused by antitrust law infringement but also its distribution among direct and indirect purchasers—a task that we set out to do in this paper.

3. Basic model

Consider a simple vertical industry structure as shown in Figure 1. There is an upstream sector with firms producing an input for the downstream sector. Note that we do not model the upstream sector. We just assume that due to cartelization or abuse of a dominant position, the upstream firms are able to raise the price w of the input to $w + dw$. The downstream firms have a cost function $c_i(q_i, w)$ which is strictly increasing and convex in q_i and increasing in w . That is, we assume that $\partial c_i(q_i, w)/\partial q_i > 0$, $\partial^2 c_i(q_i, w)/\partial q_i^2 \geq 0$, and $\partial c_i(q_i, w)/\partial w \geq 0$. Furthermore, we assume $\partial^2 c_i(q_i, w)/\partial q_i \partial w \geq 0$ where the inequality is strict for at least one firm i (otherwise $dw > 0$ does not affect the industry in the short run)¹⁶. We allow different downstream firms to have different cost functions. Some firms may simply be more efficient than others or some firms may be more dependent on the upstream firms than others. For example, some firms may have a more flexible technology that allows them to substitute away from the upstream input if w is raised. Moreover, we explicitly allow some firms not to be affected at all by the increase in w , that is we allow $\partial c_i(q_i, w)/\partial w = 0$ for some firms i . These firms may source their input outside the cartel or they may be vertically integrated with an upstream firm and therefore not directly affected by the cartel.

To start, we assume that goods produced by the downstream firms are homogeneous. Hence we can write total output as $Q = \sum_{i=1}^n q_i$ where q_i is firm i 's output level and n is the number of firms producing in the market. Downstream firms face an inverse demand function $p(Q)$, where p is strictly decreasing in Q ($p'(Q) < 0$) and $p''(Q)Q + p'(Q) < 0$ to ensure that the profit maximization problem of the firms is well defined.¹⁷

Figure 2 illustrates our basic question for the case of linear demand and costs equal to $c(q) = (c + w)q$ for each downstream firm. Due to the increase in the input price from w_0 to $w_1 = w_0 + dw > 0$, total output falls from Q_0 to Q_1 . This creates total harm for downstream firms and final consumers equal to the shaded area. We want to determine which fraction of the total harm represented in Figure 2 is harm for the downstream firms and which fraction is

¹⁶We focus here on cases where $dw > 0$ affects marginal costs and not only fixed costs. If $dw > 0$ raises firms' fixed costs, there is no price effect (for indirect purchasers) in the short run. Exit by firms can lead to higher prices in the long run. We do not analyze this case here.

¹⁷See Farrel and Shapiro (1990) for a discussion of this assumption.

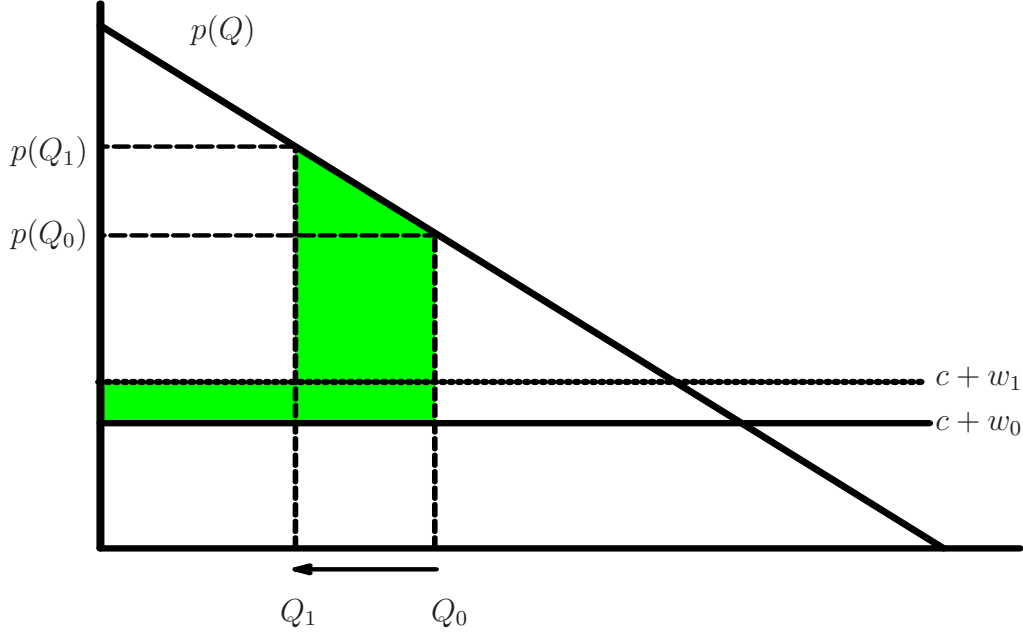


Figure 2: Total harm for downstream firms and consumers due to an increase in w leading to a fall in total output from Q_0 to Q_1 .

harm for final consumers.

To find the effect of the wholesale price w on consumer surplus $CS = \int_0^Q p(t)dt - pQ$, we differentiate CS with respect to w :

$$\frac{dCS}{dw} = -Qp'(Q)\frac{dQ}{dw} \quad (1)$$

where we use the shorthand notation $dQ/dw = \sum_{j=1}^n (dq_j/dw)$. The sign of dCS/dw is determined by the sign of dQ/dw which we determine in Lemma 1 below.

Turning to the downstream firms, we write the profit of firm i as

$$\pi_i = p(Q)q_i - c_i(q_i, w).$$

We do not want to make assumptions on the mode of competition between downstream firms. Hence we assume that firm i chooses action a_i which we normalize such that higher a_i implies higher q_i .

Then the first order condition w.r.t. a_i is given by

$$p'(Q)\frac{\partial Q}{\partial a_i}q_i + p(Q)\frac{\partial q_i}{\partial a_i} - \frac{\partial c_i(q_i, w)}{\partial q_i}\frac{\partial q_i}{\partial a_i} = 0.$$

Let

$$\theta = \frac{\partial Q}{\partial a_i} / \frac{dq_i}{da_i}, \quad (2)$$

such that θ measures the (conjectured) effect of firm i 's action on total output Q relative to i 's output.¹⁸ Hence we can write the first order condition as

$$p - \frac{\partial c_i(q_i, w)}{\partial q_i} + p'(Q)\theta q_i = 0. \quad (3)$$

Different modes of competition are nested in this framework. Firms may for example compete in price cost margins, as suggested by Grant and Quiggin (1994). Well known cases include Cournot competition with $\theta = 1$, Bertrand or perfect competition with $\theta = 0$ and the collusive outcome with $\theta = n$. From now on we work directly with equation (3) without mentioning the underlying actions a_i .

We assume that $0 \leq \theta q_i \leq Q$ for all i . The first inequality implies that firm i does not expect total output Q to fall in response to an increase in i 's action $da_i > 0$. The second inequality implies that firm i does not produce less than a monopolist (who owns all the n firms) would let firm i produce.¹⁹ Now we can prove the following result.

Lemma 1 *Assume that $\frac{\partial^2 c_i(q_i, w)}{\partial w \partial q_i} > 0$ for all i . Then an increase in w leads to a fall in total output Q . That is,*

$$\frac{dQ}{dw} < 0.$$

The intuition for this result is simple: as firms' marginal cost curves shift upward (due to an increase in w), firms reduce their output to equate marginal costs and marginal revenues again. Note that Lemma 1 means that the effect of raising the wholesale price on consumer surplus, given in equation (1), is unambiguously negative. That is, Lemma 1 implies $\frac{dCS}{dw} < 0$.²⁰ This means that in the model considered in this section consumers are always harmed to some degree if the wholesale price increases. In other words, if the downstream market produces a homogeneous good, downstream firms will always pass on some of the harm independent of the number of competitors, the form of the demand and cost functions, and the mode of competition.

Next, we are interested in the effect of w on downstream industry profits $\Pi = \sum_{i=1}^n \pi_i$. We

¹⁸Implicitly, we assume here that firms entertain symmetric conjectures. In principle, we could allow for asymmetric conjectures θ_i . However, this leads to more complicated notation while not adding much insight.

¹⁹To see this, suppose in contrast that $\theta q_i > Q$. Then equation (3) becomes

$$p - \frac{\partial c_i}{\partial q_i}(q_i, w) + p'(Q)\theta q_i < p - \frac{\partial c_i}{\partial q_i}(q_i, w) + p'(Q)Q = 0$$

where the right hand side of the inequality is a monopolist's first order condition for q_i .

²⁰If, instead, $\frac{\partial^2 c_i(q_i, w)}{\partial w \partial q_i} = 0$ for some i , the result becomes $dCS/dw \leq 0$. Consider for example the case of Bertrand competition with homogeneous goods and constant marginal costs. If the second most efficient firm's marginal costs (determining the price) are unaffected by w , then $dCS/dw = 0$.

can write

$$\begin{aligned}\frac{d\pi_i}{dw} &= p'(Q)q_i \frac{dQ}{dw} + \left(p - \frac{\partial c_i}{\partial q_i}\right) \frac{dq_i}{dw} - \frac{\partial c_i}{\partial w} \\ &= p'(Q)q_i \left[\frac{dQ}{dw} - \theta \frac{dq_i}{dw} \right] - \frac{\partial c_i}{\partial w}.\end{aligned}\tag{4}$$

Note that the second equality follows from equation (3).

The interpretation of this equation is as follows. If a firm perfectly anticipates the effect of its output level q_i on total output Q , the term in square brackets in equation (4) equals zero. The only effect left in this case is that the increase in the input price $dw > 0$ directly raises costs and therefore reduces profits (as $-\frac{\partial c_i}{\partial w} < 0$). However, in general a firm does not anticipate correctly its effect on the equilibrium level of Q . If $\left[\frac{dQ}{dw} - \theta \frac{dq_i}{dw}\right] < 0$ the firm underestimates its effect on Q and firms tend to produce too much. In this case, an increase in w which reduces both Q and q_i tends to raise downstream firms' profits. See Dixit (1986) and Quirmbach (1988) for examples where the latter effect dominates the former effect such that $d\pi_i/dw > 0$. In this case, the fall in Q raises p and therefore harms consumers. If indirect purchasers (here the final consumers) have no standing before a court, there is no incentive to sue for damages. Hence this is an example demonstrating that giving standing to indirect purchasers is important. As shown by Schinkel, Tuinstra and Rüggeberg (2005), even if $d\pi_i/dw < 0$, the upstream firms may be able to profitably compensate the downstream firms such that the latter have no incentive to sue for damages. That further makes the case that indirect purchasers should get standing. We focus on the case where indeed $d\pi_i/dw < 0$ and consider the relative harm to downstream firms and final consumers.

To prepare for the first main result of this paper, we list a few well-known terms. $e_p^Q = \frac{d \ln Q}{d \ln p}$ is the price elasticity of demand, $H = \sum_{i=1}^n \left(\frac{q_i}{Q}\right)^2$ is the Herfindahl-Hirschmann index of industry concentration, $PCM = \sum \frac{p - \partial c_i / \partial q_i}{p} \frac{q_i}{Q}$ is the industry aggregate price cost margin, $e_w^{q_i} = \frac{d \ln q_i}{d \ln w}$ is the elasticity of firm i 's output level with respect to the wholesale price w , $e_w^Q = \frac{d \ln Q}{d \ln w}$ is a similar elasticity for total production Q and z_i is the amount of the input used by firm i . Note that by Shepard's lemma we have $z_i(q_i, w) = \frac{\partial c_i(q_i, w)}{\partial w}$. With these definitions in place we can state the first main result of this paper regarding the Consumer Harm Share, CHS, which we define as the ratio of the change in consumer surplus to the change in the sum of consumer and producer surplus.

Proposition 1 *For the industry structure defined above, the consumer harm share is given by*

$$CHS := \frac{dCS/dw}{d(CS + \Pi)/dw} = \frac{1}{|e_p^Q| PCM \left(\frac{1}{H}\right) \sum_{i=1}^n \left[\left(\frac{q_i}{Q}\right)^2 \frac{|e_w^{q_i}|}{|e_w^Q|} \right] + \frac{\sum w z_i(q_i, w)}{pQ} \frac{|e_p^Q|}{|e_w^Q|}}.\tag{5}$$

Note that equation (5) is written in terms of variables that are observable or can be estimated. That is, we have substituted away the parameters θ and $\partial c_i / \partial w$ which are not readily observable. We come back to estimating these items in section 4.3.

Equation (5) says that (ceteris paribus) the consumer harm share is smaller (i) the larger is the industry aggregate price cost margin PCM , (ii) the larger is the price elasticity of demand $|e_p^Q|$ (ceteris paribus PCM), or (iii) the smaller is the wholesale price elasticity of demand e_w^Q . We provide intuition for each of these results.

First, assume that the input produced by the upstream firms is the only input used and that there is perfect competition in the downstream market such that $PCM = 0$. If it is further the case that $c(q, w) = wq$, we know that $p = w$ and $z = q$. Hence downstream firms make no profits and consumers face all the harm due to $dw > 0$. This follows immediately from equation (5) as in this case $PCM = 0$, $e_p^Q = e_w^Q$ and the income share of upstream firms $\frac{\sum w z_i(q_i, w)}{pQ}$ equals 1 and thus $CHS = 1$.

Now assume that the elasticities satisfy $e_p^Q = e_w^Q$ and $e_{q_i}^Q = e_w^Q$ for all i . That is, a one percent increase in w leads to the same percentage fall in the equilibrium level of Q as a one percent increase in p . Further, a one percent increase in w decreases each firm's output level q_i (and therefore total output Q) with the same percentage. In this case the CHS given by (5) can be written as

$$CHS = \frac{1}{|e_p^Q|PCM + \frac{\sum w z_i(q_i, w)}{pQ}}. \quad (6)$$

The first term in the denominator is related to the pass through term and the second term is the cost effect for the downstream firms. The higher is the PCM, the more the increase in w will be absorbed by the firms and the lower the harm that will be passed on to consumers. Or put differently, the lower is PCM, the less the firms will absorb. The higher the price elasticity of demand (for given $PCM > 0$), the harder it is for firms to raise their price (in response to $dw > 0$) and hence firms bear more of the harm. Finally, the second term in the denominator of equation (6) shows that the higher the income share of the input (ceteris paribus the pass on), the more harmful an increase in w for the downstream firms. Clearly, if the input is only 1% of total revenue, the price increase $dw > 0$ (ceteris paribus the pass through) hardly raises costs and is not going to hurt downstream firms much.

Finally, going back to equation (5) there are two effects that have not yet been discussed. For given $|e_p^Q|$, the smaller is $|e_w^Q|$, the less equilibrium output responds to $dw > 0$. Hence the more harm is absorbed by firms' PCM and hence the higher the part of the harm borne by the firms. Finally, the term

$$\frac{\sum_{i=1}^n \left(\frac{q_i}{Q}\right)^2 \frac{|e_{q_i}^Q|}{|e_w^Q|}}{H} \quad (7)$$

can be seen as a weighted average of $\frac{|e_{q_i}^Q|}{|e_w^Q|}$ where the weights equal firm's squared market shares (since $H = \sum_{i=1}^n \left(\frac{q_i}{Q}\right)^2$). If big firms are relatively less responsive to a change in w than small firms, the expression in equation (7) is relatively small and consumers tend to bear more of the harm due to $dw > 0$. The reason is as follows. As w increases, firms' outputs are reduced (see lemma 1). If this happens to a smaller extent for big firms than for small ones (because the

big firms are less responsive), then concentration will increase. This increase in concentration raises market power which leads to higher prices. This raises CHS. To see the relation between the term in (7) and the effect of w on H more clearly, we write

$$\begin{aligned}\frac{d \ln H}{d \ln w} &= \frac{2w}{H} \sum \frac{q_i}{Q} \left(\frac{1}{Q} \frac{dq_i}{dw} - \frac{q_i}{Q^2} \frac{dQ}{dw} \right) \\ &= \frac{2}{H} e_w^Q \sum \left(\frac{q_i}{Q} \right)^2 \left(\frac{e_w^{q_i}}{e_w^Q} - 1 \right) \\ &= 2e_w^Q \left(\frac{\sum_{i=1}^n \left(\frac{q_i}{Q} \right)^2 \frac{|e_w^{q_i}|}{|e_w^Q|}}{H} - 1 \right)\end{aligned}$$

We see that $\frac{d \ln H}{d \ln w} = 0$ if $\frac{1}{H} \sum_{i=1}^n \left(\frac{q_i}{Q} \right)^2 \frac{|e_w^{q_i}|}{|e_w^Q|} = 1$. If all firms react to the same extent to $dw > 0$, there is no effect on concentration. If bigger firms are less responsive, we find that $\frac{1}{H} \sum_{i=1}^n \left(\frac{q_i}{Q} \right)^2 \frac{|e_w^{q_i}|}{|e_w^Q|} < 1$ and H increases in response to $dw > 0$ (because $e_w^Q < 0$). This increase in market power leads to higher prices, thereby increasing CHS.

When faced with the task of determining the distribution of harm, the practitioner can in general proceed in two different ways. First, one can use equation (5) and directly estimate all necessary terms given in this equation. This is what we illustrate in section 4.3. Second, one can make specific parametric assumptions on demand and costs and see whether this reduces the number of terms that need to be estimated. The latter approach is what we illustrate next.

Example 1 (*Linear demand and costs*) Let inverse demand be given by $P(Q) = a - bQ$ and assume that costs are given by $C_i(q_i, w) = (c_i + w)q_i$. In this case $CHS = \frac{n}{n+2\theta}$. Note that CHS is independent of the size of the market (a), marginal production costs (c_i), and the wholesale price (w). The only item to be estimated is the conjectural variation θ which can be determined using equation (A.3) in the appendix. Note furthermore that $CHS = 1$ if either $n \rightarrow \infty$ or $\theta = 0$ (Bertrand). In these cases all harm is completely passed on to consumers. Further, $CHS = 1/3$ when $\theta = 1$ and $n = 1$ (Monopoly)²¹ or when $\theta = n$ (Collusion). In general, it holds that $1/3 \leq CHS \leq 1$.

Example 2 (*Constant-elasticity demand and linear costs*) Let inverse demand be given by $P(Q) = a^{(1/b)} Q^{-(1/b)}$ ($Q(P) = aP^{-b}$) and assume that symmetric costs are given by $C_i(q_i, w) = (c + w)q_i$. In this case we have $CHS = \frac{bn}{bn - \theta + b\theta}$. Thus we get $CHS = 1$ if either $n \rightarrow \infty$ or $\theta = 0$ (Bertrand). Further, $CHS = b/(2b - 1)$ when $\theta = 1$ and $n = 1$ (Monopoly) or when $\theta = n$ (Collusion). Furthermore, we can have $CHS > 1$ if $b < 1$ and $\theta > 0$. This result is due to the fact that in this case firms' profits increase rather than decrease with a rising wholesale price w over this range of the demand elasticity b . See e.g. Seade (1985).

²¹This can be viewed as the harm counterpart to the known result that the *price* pass through rate (that is, the change in the price charged to consumers relative to the change in marginal costs stemming e.g. from the imposition of a unit tax) is exactly 50 percent if a monopolist faces linear demand and constant marginal costs. See, e.g., Kosicki and Cahill (2006), p.612.

Note that in both of these examples CHS is independent of the wholesale price w implying that for the determination of CHS the “but for” price is not needed.

In these two examples we assume that all firms are affected. But actually this is not necessary for equation (5) to hold. Even if only a subset of firms is affected by the increase in w , the distribution of harm is still given by (5). We illustrate this by considering the case where all firms face the same cost function $c(q, w)$. In particular, out of the n firms, $m \in \{1, \dots, n-1\}$ face a price increase $dw > 0$. Although, it follows directly from proposition 1 that CHS is not affected, we also provide a direct proof to illustrate this result.

Corollary 1 *In the case where firms produce homogeneous goods and where $n - m$ firms have a cost function $c(q, w)$ while m firms have a cost function $c(q, w + dw)$ it holds that*

$$\frac{dCHS}{dm} = 0.$$

3.1. Differentiated products

Instead of assuming homogeneous goods as above, here we allow goods to be differentiated. In particular, we assume the utility function of a representative consumer takes the form $u(q_1, \dots, q_n) + x$ with some outside good x (sold at a normalized price equal to 1). By maximizing consumer surplus $u(q_1, \dots, q_n) + y - \sum_{i=1}^n p_i q_i$ (where y denotes the amount of money the consumer wants to spend this period), the inverse demand curve for firm i is given by

$$p_i(q_i, q_{-i}) = \frac{\partial u}{\partial q_i}.$$

Firm i ’s own demand elasticity is defined as

$$e_{p_i}^{q_i} = \frac{\partial q_i}{\partial p_i} \frac{p_i}{q_i}. \quad (8)$$

We focus here on the symmetric case where firms have the same cost functions $c(q, w)$, face symmetric demand functions and play a symmetric equilibrium.²² We define the (market)

²²This symmetry assumption is necessary to get a straightforward definition of the market demand elasticity. We do not know how to meaningfully define a market demand elasticity in case firms charge different prices and produce different output levels. Then a one percent increase in each firm’s price can lead to different percentage changes in firms’ output levels. Since goods are differentiated we cannot simply add these output levels (adding “apples and oranges”). If in a particular case, the symmetry assumption is clearly violated, equation (5) can be applied by assuming that each firm acts as a (local) monopolist on the market of its own (differentiated) product. This is, of course, always possible but is more demanding on the time-series dimension of the data as firm specific variables cannot be estimated on the cross section of firms (unless one is willing to make additional assumptions).

demand elasticity e_p^q as follows. Differentiating the (inverse) demand function for firm i we can write:

$$d \ln p_i = \sum_{j=1}^n \frac{\partial p_i}{\partial q_j} \frac{q_j}{p_i} d \ln q_j.$$

We consider a symmetric equilibrium where all prices p_i increase with the same percentage $d \ln p$. As a consequence all output levels change with the same percentage as well, denoted by $d \ln q$. Then we define the market elasticity, e_p^q , as the percentage change in output as the result of a 1% increase in all prices:

$$e_p^q = \frac{d \ln q}{d \ln p} = \frac{1}{\sum_{j=1}^n \frac{\partial p_i}{\partial q_j} \frac{q_j}{p_i}}. \quad (9)$$

Note that e_p^q is the elasticity of Chamberlin's DD curve that traces out the quantity demanded from firm i when all firms' prices change.

Consumer surplus is defined as

$$CS = u(q_1, \dots, q_n) + y - \sum_{j=1}^n p_j q_j.$$

Hence we find

$$\frac{dCS}{dw} = - \sum_{i=1}^n q_i \sum_{j=1}^n \frac{\partial p_i}{\partial q_j} \frac{dq_j}{dw}.$$

In a symmetric equilibrium, we can write this as

$$\frac{dCS}{dw} = np \frac{1}{|e_p^q|} \frac{dq}{dw} < 0. \quad (10)$$

Profits of firm i are defined as

$$\pi_i = p(q_i, q_{-i})q_i - c(q_i, w).$$

The first order condition can be written as

$$\frac{\partial p_i}{\partial q_i} \frac{q_i}{p_i} p_i (1 - \theta) + \sum_{j=1}^n \frac{\partial p_i}{\partial q_j} \frac{q_j}{p_i} \frac{p_i}{q_j} \theta q_i + p_i - \frac{\partial c}{\partial q_i} = 0$$

where $\theta = dq_j/dq_i$ for $i \neq j$.²³ In a symmetric equilibrium this can be written as

$$\frac{p(1-\theta)}{|e_{p_i}^q|} + \frac{p\theta}{|e_p^q|} = p - \frac{\partial c}{\partial q}. \quad (11)$$

To see the effect of w on industry profits, write first

$$\frac{d\pi_i}{dw} = \sum_{j=1}^n \frac{\partial p}{\partial q_j} \frac{dq_j}{dw} + \frac{dq_i}{dw} p - \frac{\partial c}{\partial q_i} \frac{dq_j}{dw} - \frac{\partial c}{\partial w} \quad (12)$$

Using (11), we can write

$$\frac{d\pi}{dw} = n \left[p(1-\theta) \frac{dq}{dw} \left(\frac{1}{|e_{p_i}^q|} - \frac{1}{|e_p^q|} \right) - \frac{\partial c}{\partial w} \right]. \quad (13)$$

From this we can derive the following result.

Proposition 2 *If symmetric firms produce differentiated products, we find that the distribution of the total harm due to $dw > 0$ is distributed over downstream firms and final consumers as follows*

$$CHS = \frac{1}{|e_p^q|PCM + \frac{|e_p^q|}{|e_w^q|} \frac{nz w}{npq}}. \quad (14)$$

Analogously, to the case of homogeneous products, equation (14) says that (ceteris paribus) the consumer harm share is smaller (i) the larger is the industry aggregate price cost margin PCM , (ii) the larger is the price elasticity of demand $|e_p^q|$ (for given PCM), or (iii) the smaller is the wholesale price elasticity of demand $|e_w^q|$.

Note that the expression in equation (14) is the same as in equation (5) for the case where firms are symmetric. The difference is that firms producing differentiated products tend to face a demand function that is less elastic.

Example 3 (*Linear demand and costs*) Let inverse demand be given by $p_i(q) = \alpha - \beta q_i - \gamma \sum_{j \neq i} q_j$ where $\beta > \gamma > 0$ and $\alpha > 0$, and assume that costs are given by $C(q_i, w) = (c + w)q_i$. Straightforward computations yield $CHS = \frac{\beta + \gamma(n-1)}{3\beta + \gamma(2\theta + 1)(n-1)}$.²⁴ Again, the CHS is independent of the size of the market (α), marginal production costs (c), and the wholesale price (w). The only items to be estimated are the conjectural variation θ and the demand parameters β and γ .

²³Note that in contrast to the case of a homogeneous good treated in section 3, θ here measures the effect of firm i 's quantity on firm j 's quantity (not on the total quantity Q which is not defined with differentiated goods).

²⁴For $\beta = \gamma$ this expression is the same as in example 1. To verify this, note that θ with differentiated goods is defined as $\theta^d = \partial q_j / \partial q_i$ for $j \neq i$, while for homogeneous goods we define it as $\theta^h = \partial Q / \partial q_i$. Hence for the symmetric case here, we have $\theta^d = (\theta^h - 1)/(n - 1)$.

Example 4 (*Constant-elasticity demand and linear costs*) Let inverse demand be given by $p_i(q) = \beta\gamma \left(\sum_{j=1}^n q_j^\beta\right)^{\gamma-1} q_i^{\beta-1}$ and assume that $0 < \beta\gamma < 1$ (for strict concavity), $\gamma < 1$, and $\beta \leq 1$. If $\beta > 1$, $\beta = 1$, $\beta < 1$, goods are partial substitutes, homogeneous, complements. Again, assume that costs are given by $C(q_i, w) = (c + w)q_i$. In this case we have

$$CHS = -\frac{\beta n(1 - \gamma)}{n - \beta^2\gamma(\gamma + n + \theta - 1 + \theta(\gamma(n - 1) - n))}.$$

4. Empirically estimating the harm

The *CHS* expression consists of two types of variables: elasticities and other variables. In this section we describe how the values of these variables can be estimated. First, we discuss the elasticities. An important elasticity measures the effect of w on output of downstream firms. As argued below, if w is raised due to a cartel, there is reason to believe that w will be relatively constant over time. This might make it hard to estimate its elasticity. Hence we first describe how demand shifts can be used as well to infer the elasticity. This leads to the concept of a supply curve in oligopoly. Then we proceed by describing the data needed and the method to estimate the elasticities and other variables.

4.1. What if w has hardly changed over time?

As suggested by studies on cartels (*e.g.* Harrington (forthcoming) and references therein), if the upstream firms form a cartel, there may be a tendency to keep w fairly constant over time (even though the costs of producing the downstream input for the upstream firms does vary over time). This can make it problematic to identify the elasticities $e_w^{q_i}, e_w^Q$ in equation (5).²⁵ If this is the case, one can use another input for downstream firms and see how changes in the price of the alternative input affects q_i and Q . If these inputs are used in fixed proportions, then using this method is perfectly fine. If there is some room for substitution between the inputs, this method can be seen as an approximation.

If such an alternative input is not available, one can also use shifts in demand as a way to get information on the effects of cost shifts. This approach is illustrated here. An additional advantage of this approach is that using demand shocks allows us to derive a supply curve for oligopoly. Using this supply curve, we can generalize the following well known result in the tax incidence literature. The harm of a tax per unit of output introduced by the government is distributed over producers and consumers depending on the relative slope of the demand and supply curves.

Let us again consider a market for a homogeneous good. We consider the following perturbation of the inverse demand function

$$p(Q) + \varepsilon$$

²⁵Note that a low variance in w over time does not complicate the estimation of the other factors in (5) such as the income share of the input.

where ε is thought to be small and either positive or negative. Hence changes in ε lead to parallel shifts of the inverse demand function as illustrated in Figure 3. The first order condition for q_i can now be written as

$$p + \varepsilon - \frac{\partial c_i}{\partial q_i} + p'(Q)\theta q_i = 0. \quad (15)$$

Hence a demand shift is identical to a shift in w if

$$d\varepsilon = -\frac{\partial^2 c_i}{\partial q_i \partial w} dw.$$

To ease notation, we focus here on the case where cost functions take the form $c_i(q, w) = wq + c_i(q)$. Then equivalent changes satisfy $d\varepsilon = -dw$. In this case, we can use equation (5) with $|e_w^{q_i}| = e_\varepsilon^{q_i}$, $|e_w^Q| = e_\varepsilon^Q$. That is, we identify these elasticities using demand shifts instead of changes in costs.

4.2. Tax incidence intuition

To see the equivalence between our approach on the distribution of harm and the results in the tax incidence literature, we define a supply curve for oligopoly in the following way. With $p(Q) + \varepsilon$, changes in ε will generate different equilibrium combinations for p and Q . Mapping out these points $(Q(\varepsilon), p(\varepsilon))$, as in Figure 3, gives what we call the (oligopoly) supply curve.²⁶ Under perfect competition, in equilibrium price equals marginal costs. Thus the curve created in this way is the marginal cost curve of the sector, which is indeed the supply curve as it is used in, for instance, the tax literature. Then we know that the slopes of the marginal costs and demand curves determines the incidence. We define the slope, ψ , of the (oligopoly) supply curve as:

$$\psi = \left. \frac{dp}{dQ} \right|_{supply} = \frac{dp/d\varepsilon}{dQ/d\varepsilon} = \frac{p'(Q) \frac{dQ}{d\varepsilon} + 1}{\frac{dQ}{d\varepsilon}}. \quad (16)$$

Using that the cost function above implies $z_i = q_i$, equation (5) can be rewritten as follows:

$$\frac{dCS/dw}{d(CS + \Pi)/dw} = \frac{1}{|e_p^Q| PCM \frac{1}{H} \sum_{i=1}^n \left[\left(\frac{q_i}{Q} \right)^2 \frac{|e_w^{q_i}|}{|e_w^Q|} \right] + 1 + \frac{\psi}{-p'}}. \quad (17)$$

Under perfect competition, we have that $PCM = 0$ and thus $CHS = -\frac{p'}{\psi - p'}$. Hence we replicate the result that the incidence of harm due to $dw > 0$ is determined by the relative slopes of

²⁶Note the difference between the supply curve defined in this way and a supply relation as defined in the literature (e.g., in equation (4) in Bresnahan (1989)). In the literature a supply relation usually is the first-order condition of profit maximization as in equation (15) above. (The sum of the first-order conditions for all firms is referred to as industry supply.) However, we refer to a supply curve as the locus of equilibrium combinations for p and Q in reaction to the change in a demand shifter.

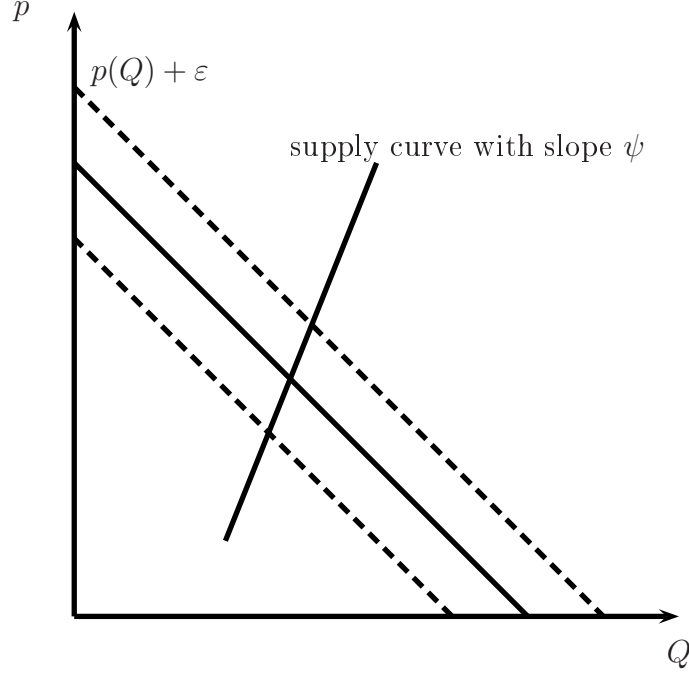


Figure 3: Demand shifts ε plot out the (oligopoly) supply curve.

demand and supply.²⁷ Under oligopoly, however, there is an additional term as $PCM > 0$, but the main intuition from the tax literature applies here as well. The steeper the slope of the supply curve relative to the demand curve, the more downstream firms bear the harm relative to final consumers.

To get some idea of what determines the slope ψ , consider equation (15) for the case of symmetric firms and differentiate with respect to ε . Then one can verify that

$$\frac{dQ}{d\varepsilon} = \frac{n}{-p''(Q)Q\theta - p'(Q)(n + \theta) + \frac{\partial^2 c}{\partial q^2}}.$$

Substituting this into equation (16) yields

$$\psi = -\frac{1}{n} \left(\theta p''(Q)Q + \theta p'(Q) - \frac{\partial^2 c}{\partial q^2} \right) \quad (18)$$

Hence higher n and lower θ (for given Q) lead to a flatter supply curve. Thus, the more firms there are on the market and the more aggressive their conduct is (lower θ) the flatter the supply curve. In this case, the firms do not absorb the increase $dw > 0$ and hence consumers bear a bigger fraction of the harm.

²⁷ Compare this to the familiar formula $\Delta P / \Delta MC = e_p^S / (e_p^S - e_p^D)$ of the change of the consumer price (ΔP) relative to the change in marginal costs (ΔMC) following the imposition of a unit tax in a competitive market. Here, e_p^S (e_p^D) denotes the price elasticity of supply (demand). See, e.g., Pindyck and Rubinfeld (2005), p.326.

4.3. Specifics on the empirical estimation of the distribution of harm

Finally, we illustrate how the harm distribution can be estimated in practice. We do this for the homogeneous good case. It is easy to adjust this for the heterogeneous good case.

In a typical abuse case, one has available (or can relatively easily get) the following information for the firms in the relevant market: output per firm, the input price causing the harm, the amount of the input used per firm, other costs and cost shifters, price of the downstream firms' output and demand shifters. We need to have this information for a couple of periods t (usually years). Let us consider each in turn.²⁸

It should be relatively easy to get the information on the downstream firms' output levels q_{it} as they may actually be bringing the case and in that sense should be expected to cooperate. Also, information on output is relatively easy to verify. With this information, we can calculate total output $Q_t = \sum q_{it}$ per period as well. The input price causing the harm, is here denoted by w_{0t} . Information on other input prices is denoted by w_{jt} . To calculate PCM we need information on marginal costs. That is usually hard to get and one can use average variable costs as an approximation. We only need PCM on the industry level. This can be approximated by operating profits divided by sales, where operating profits are defined as sales minus material and payroll costs (Aghion, Bloom, Blundell, Griffith and Howitt (2005) and Scherer and Ross (1990)).

The price of downstream firms' output is denoted by p_t . Demand shifters include consumers' income and changes in demand for complementary goods. For instance, in Porter (1983) a demand shock for a US railroad cartel is indentified by whether or not the shipping routes on the Great Lakes were free of ice. We denote demand shifters by y_{kt} . Finally, we need to know total expenditure on the input, $w_{0t}Z_{0t}$

With this information we calculate

$$\begin{aligned} PCM_t &= \frac{\text{operating profit}_t}{\text{sales}_t} \\ H_t &= \sum_i \left(\frac{q_{it}}{Q_t} \right)^2 \\ \frac{\sum w_{0t}z_{i0t}}{p_t Q_t} &= \frac{w_{0t}Z_{0t}}{p_t Q_t} \end{aligned}$$

Now turning to the elasticities, we rewrite equation (5) which will allow us to use slightly simpler estimation techniques.²⁹ In particular, note that

$$\frac{|e_p^Q|}{|e_w^Q|} = \frac{\frac{d \ln Q}{d \ln p}}{\frac{d \ln Q}{d \ln w}} = \frac{1}{e_w^p}$$

²⁸Note that in many countries this type of firm level data is present at the national statistical office. There it is used for the country's national accounts.

²⁹By not estimating the price elasticity e_p^Q , we do not need to deal with the endogeneity of p when estimating its effect on Q .

where

$$e_w^p = \frac{d \ln p}{d \ln w}$$

is the (equilibrium) elasticity of the final output price p with respect to the input price w . Using this we write (5) as follows.

$$CHS = \frac{e_w^p}{PCM \left(\frac{1}{H} \right) \sum_{i=1}^n \left[\left(\frac{q_i}{Q} \right)^2 |e_w^{q_i}| \right] + \frac{\sum w z_i(q_i, w)}{pQ}}.$$

To determine the elasticities e_w^p and $e_w^{q_i}$ we run the following regressions.³⁰

$$\begin{aligned} \ln p_t &= \alpha_0 + \alpha_{w0} \ln w_{0t} + \sum_j \alpha_{wj} \ln w_{jt} + \sum_k \alpha_{yk} \ln y_{kt} + \varepsilon_t \\ \ln q_{it} &= \beta_{i0} + \beta_{wi0} \ln w_{0t} + \sum_j \beta_{wji} \ln w_{jt} + \sum_k \alpha_{yki} \ln y_{kt} + \varepsilon_{it} \end{aligned}$$

The first equation allows one to identify the price elasticity as $e_w^p = \alpha_{w0}$. The second equation can be estimated for each firm separately or as a panel if certain elasticities are assumed to be the same across firms. The relevant elasticity can then be identified as $e_w^{q_i} = \beta_{wi0}$.

When estimating these equations, there can be an endogeneity problem with w_{0t} on the right hand side. In particular, if demand in the downstream market shifts out, p_t tends to increase and demand for the input goes up. If upstream firms face increasing marginal costs, w_{0t} will increase as well. This leads to a biased estimate of α_{w0} . Under either of the following two conditions one does not need to worry about this endogeneity bias. First, if all relevant demand shifts in the downstream market are picked up by the demand shifters y_{kt} variables. Second, if the downstream sector under consideration is one of many sectors buying the input from the upstream sector and upstream firms are not able to price discriminate between firms from different sectors. In this case, it is unlikely that shifts in this downstream market affect w_{0t} . Hence, the variation in w_{0t} is then caused by exogenous cost shifts for the upstream firms.

If neither of these conditions holds, one needs to collect data on upstream cost shifts. These are then used to instrument w_{0t} . The instrumented wholesale price is then used to estimate α_{w0} and β_{wi0} in the equations above.

If w_{0t} does not vary enough over time, it becomes hard to estimate α_{w0} and β_{wi0} . If there is no or not enough variation in w_{0t} , one can choose another input j which is used in a similar way as the input under consideration and then one can approximate $e_w^p = \alpha_{wj}$. If such an input is not available, one can use one of the demand shifters y_{kt} , as described in the previous sub-section to identify the elasticities.

³⁰Note the similarity with the Panzar-Rosse statistic (Panzar and Rosse (1987)) defined as the sum of factor price elasticities of firms' revenues or output levels.

5. Summary and concluding remarks

One of the reasons why the U.S. Supreme Court ruled out a pass-on defence in the 1968 landmark case *Hanover Shoe* was that the task of showing the extent of pass on “would normally prove insurmountable.” In fact, forty years after this ruling Bulst (2006, p. 738) states that: “There seems to be no reported court decision, neither in the United States, the United Kingdom, France nor Germany, in which a court calculated or estimated the amount of an overcharge passed on to an intermediate purchaser.”

In this paper we suggest a general framework that allows to determine how the total harm due to *e.g.* price-fixing in an upstream market is distributed over firms in a downstream market and final consumers. In this framework we make no specific assumptions regarding demand, costs, the mode of competition, or the kind of production technology that downstream firms use in order to turn inputs into final consumer goods. We show how the consumer harm share can be determined both when goods produced downstream are homogeneous or differentiated. Furthermore, we develop a procedure that allows to estimate the relevant terms for the harm distribution even if elevated upstream prices are rather constant over time. Finally, we sketch how a practitioner can actually estimate the relevant items in the expression of the consumer harm share.

The motivation for this exercise is two-fold. First, with the framework we put forward here we hope to contribute to showing that in principle the task of apportioning antitrust harm in vertically related industries is not “insurmountable”—an assessment that was perhaps never shared by all economic observers. We see this as complementary to recent efforts of reconsidering the determination of the absolute amount of harm resulting from anti-competitive price-fixing cases as put forward in *e.g.* Hellwig (2006), Verboven and Dijk (2007), and Basso and Ross (2007).

Second, not allowing a pass-on defence may create unjustified windfall profits for direct purchasers as they can claim the entire overcharge even if they passed on some or all of this overcharge to their customers. Van Dijk and Verboven (2005) hint at the possibility that this may lead to distorted prices. Moreover, in the 1977 *Illinois Brick* ruling, indirect purchasers were denied the right to sue for antitrust damages. This implies the problem that parties who were harmed cannot sue for compensation. Due to these problems, the two court rulings of *Hanover Shoe* and *Illinois Brick* have attracted a lot of criticism.³¹ In response, changes in the law have already been established (such as the *Illinois Brick* repealers) while others are likely to be implemented in the future (see *e.g.* the suggestions of the Antitrust Modernization Committee as cited in section 2). This creates a sense of urgency to develop methods for the practical apportionment of harm over the various links in a production/supply chain. With this paper we hope to make a contribution towards this goal.

We end this paper with some remarks.

³¹Of course there are several reasons in favor of ruling against a pass-on defence and against indirect purchasers to have standing as put forward by *e.g.* Landes and Posner (1979). Among those reasons are that direct purchasers might have an informational advantage due to their closeness to the infringer, and that indirect purchasers might have small and dispersed claims which lessens their incentives to sue for damages.

First, in the models above we only assumed that the upstream sector “somehow” manages to illegally increase the wholesale price w . Hence, our analysis does not only apply to plain price-fixing agreements, but to all kinds of anticompetitive strategic behavior that result in an elevated wholesale price such as (input) foreclosure, predatory pricing (after having been successful), limit pricing or exclusive dealing.

Second, our results equally apply to the question of how cost savings upstream (due to, say, merger) are passed on to downstream firms and consumers. For a related discussion see Ten Kate and Niels (2005).

Third, in our analysis we did not consider the possibility that the unlawful rise in the upstream price may lead to adjustment by firms in the form of entry or exit. We leave this as a topic for future research. We note, however, that the practitioner faced with the task of estimating the consumer harm share given in equations (5) and (14) could use long-run instead of short-run elasticities to take this into account.

Fourth, for simplicity our analysis above assumed an industry structure consisting of only three layers. However, it is conceivable that the production or supply chain consists of more than three layers.³² If this is the case, the *CHS* developed in this paper can be applied several times to determine the share of the total harm that is borne by each layer of the industry. For example, let’s assume that there are four layers: an upstream sector (U), two consecutive downstream sectors (D_1 and D_2), and final consumers (C). Furthermore, assume that the upstream sector charges the illegally raised wholesale price w to downstream sector D_1 , which in turn increases the price p_1 it charges to firms in the downstream sector D_2 , which in turn increases the final consumer price p_2 . In this case, one can use our framework computing two consumer harm shares. The first (CHS_1) only considering the chain $U - D_1 - D_2$ and substituting the final consumer demand we used in our analysis above with the demand function of downstream sector D_2 . The second (CHS_2) considering the chain $D_1 - D_2 - C$ where D_1 takes the role of the upstream sector raising price p_1 . Note that CHS_1 can be used as a screening device for how severe the pass-on from the upstream sector down the production chain really is. If CHS_1 is “sufficiently small,” then the entire case can be dismissed and there would be no need to determine CHS_2 . If CHS_1 turns out to be “sufficiently big,” however, one can use the two *CHSs* to determine the share of the total harm that is borne by each layer in the chain.

³²Note again, that Han, Schinkel and Tuinstra (2008) consider a model with an arbitrary number of layers.

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Appendix A. Proofs

Proof of lemma 1: Differentiate equation (3) with respect to w as follows

$$[p'(Q) + p''(Q)\theta q_i] \frac{dQ}{dw} + \left(p'(Q)\theta - \frac{\partial^2 c_i}{\partial q_i^2} \right) \frac{dq_i}{dw} = \frac{\partial^2 c_i}{\partial q_i \partial w}.$$

The assumptions $p'(Q) < 0$, $p''(Q)Q + p'(Q) < 0$ and $0 \leq \theta q_i \leq Q$ imply that the term in square brackets is negative. Further, the assumption $\frac{\partial^2 c_i}{\partial q_i^2} \geq 0$ implies that the second term in brackets is negative as well. Next, $\frac{\partial^2 c_i}{\partial q_i \partial w} > 0$ implies $dQ/dw < 0$ for the following reason. Suppose by contradiction that $dQ/dw \geq 0$, then we find $dq_i/dw < 0$ for all i . However, since $Q = \sum_i q_i$ this leads to a contradiction. *Q.E.D.*

Proof of proposition 1: Summing equation (4) over all i yields

$$\frac{d\Pi}{dw} = p'(Q)Q \frac{dQ}{dw} \left(1 - \theta \sum \frac{q_i}{Q} \frac{dq_i/dw}{dQ/dw} \right) - \sum \frac{\partial c_i}{\partial w}. \quad (\text{A.1})$$

Hence, comparing the loss in profits to the loss in consumer surplus (CS), given in (1), we get

$$\frac{d\Pi/dw}{dCS/dw} = \theta \sum \frac{q_i}{Q} \frac{dq_i/dw}{dQ/dw} + \frac{\sum \frac{\partial c_i}{\partial w}}{Qp'dQ/dw} - 1. \quad (\text{A.2})$$

Now, write equation (3) as

$$\frac{p - \frac{\partial c_i}{\partial q_i}}{p} = -\frac{dp}{dQ} \frac{Q}{p} \theta \frac{q_i}{Q}$$

and multiply both sides of this equation by q_i/Q to get

$$PCM = \sum_i \frac{q_i}{Q} \frac{p - \frac{\partial c_i}{\partial q_i}}{p} = \frac{\theta H}{|e_p^Q|}$$

or

$$\theta = \frac{|e_p^Q|}{H} PCM. \quad (\text{A.3})$$

Using (A.3), rewrite equation (A.2) as

$$\frac{d\Pi/dw}{dCS/dw} = \frac{|e_p^Q|}{H} PCM \sum \left(\frac{q_i}{Q} \right)^2 \frac{\frac{dq_i}{dw} \frac{w}{q_i}}{\frac{dQ}{dw} \frac{w}{Q}} + \frac{\frac{\sum w \frac{\partial c_i}{\partial w}}{pQ}}{p' \frac{Q}{p} \cdot \frac{dQ}{dw} \frac{w}{Q}} - 1. \quad (\text{A.4})$$

Using Shepard's lemma $\frac{\partial c_i}{\partial w}(q_i, w) = z_i(q_i, w)$, equation (A.4) is equivalent to

$$\frac{d\Pi/dw}{dCS/dw} = \frac{|e_p^Q|}{H} PCM \sum \left(\frac{q_i}{Q} \right)^2 \frac{|e_w^{q_i}|}{|e_w^Q|} + \frac{\frac{\sum w z_i(q_i, w)}{pQ}}{|e_w^Q|} |e_p^Q| - 1. \quad (\text{A.5})$$

Hence

$$\frac{d(\Pi + CS)/dw}{dCS/dw} = \frac{d\Pi/dw}{dCS/dw} + 1 = \frac{|e_p^Q|}{H} PCM \sum \left(\frac{q_i}{Q} \right)^2 \frac{|e_w^{q_i}|}{|e_w^Q|} + \frac{\frac{\sum w z_i(q_i, w)}{pQ}}{|e_w^Q|} |e_p^Q|.$$

From this the equation in the proposition follows. Q.E.D.

Proof of Corollary 1: Profits for *affected* and *unaffected* firms are given by, resp.

$$\pi_u = p(Q)q_u - c(q_u, w) \quad (\text{A.6})$$

$$\pi_a = p(Q)q_a - c(q_a, w + dw). \quad (\text{A.7})$$

The first order condition for a firm $i = a, u$ can be written as

$$p(Q) - c'_{q_i} + p'(Q)\theta q_i = 0$$

and total output is given by

$$Q = (n - m)q_u + mq_a.$$

The effect of dw (evaluated at $dw = 0$) on total industry profits can now be written as

$$\frac{d\Pi}{dw} = Qp'(Q)\frac{dQ}{dw} + (P(Q) - c'_q)\frac{dQ}{dw} - mc'_w \quad (\text{A.8})$$

where we can write $c'_q = c'_{q_a} = c'_{q_u}$ precisely because we evaluate at $dw = 0$. To find the effect of dw on Q we differentiate the first order conditions for q_a, q_u with respect to w to get

$$-SOC \frac{dq_u}{dw} = [p'(Q) + p''(Q)\theta q_u] \frac{dQ}{dw} \quad (\text{A.9})$$

and

$$-SOC \frac{dq_a}{dw} = [p'(Q) + p''(Q)\theta q_a] \frac{dQ}{dw} - c''_{wq_a} \quad (\text{A.10})$$

where $SOC = 2p'(Q)\theta - c''_{qq} + p''(Q)\theta^2 q < 0$ stands for the second order condition. Multiply equation (A.9) by $n - m$ and equation (A.10) by m , then add the two equations to get

$$[-SOC - np'(Q) - p''(Q)\theta Q] \frac{dQ}{dw} = -mc''_{qw}$$

Put differently, $\frac{dQ}{dw}$ is linear in m . Using $\frac{dCS}{dw} = -Qp'(Q)\frac{dQ}{dw}$, we find

$$\frac{d(CS + \Pi)/dw}{dCS/dw} = -\frac{P(Q) - c'_q}{Qp'(Q)} + \frac{c'_w}{Qp'(Q)} \frac{m}{dQ/dw}$$

which is independent of m because—as found above— dQ/dw is linear in m . Hence, also CHS (the reciprocal of $\frac{d(CS+\Pi)/dw}{dCS/dw}$) is independent of m . *Q.E.D.*

Proof of Proposition 2: First, using (10) and (13) we get

$$\frac{d\Pi/dw}{dCS/dw} = \frac{-(1-\theta)\frac{dq}{dw}\left(\frac{1}{|e_p^q|} - \frac{1}{|e_{p_i}^{q_i}|}\right) - \frac{z}{p}}{\frac{dq}{dw}\frac{1}{|e_p^q|}} \quad (\text{A.11})$$

$$= -(1-\theta) \left[1 - \frac{|e_p^q|}{|e_{p_i}^{q_i}|} \right] + \frac{|e_p^q|}{|e_w^q|} \frac{nz w}{npq} \quad (\text{A.12})$$

Next, from (11) we find

$$PCM = \sum_{i=1}^n \frac{q}{Q} \frac{p - c'_q}{p} = (1-\theta) \frac{1}{|e_{p_i}^{q_i}|} + \frac{\theta}{|e_p^q|} \quad (\text{A.13})$$

or

$$\theta = \left(\frac{1}{|e_{p_i}^{q_i}|} - PCM \right) \frac{|e_{p_i}^{q_i}| |e_p^q|}{|e_p^q| - |e_{p_i}^{q_i}|}. \quad (\text{A.14})$$

Substituting this expression for θ into (A.12) leads to

$$\frac{d\Pi/dw}{dCS/dw} = -1 + |e_p^q| PCM + \frac{|e_p^q|}{|e_w^q|} \frac{nz w}{npq} \quad (\text{A.15})$$

Finally, from $\frac{d(\Pi+CS)/dw}{dCS/dw} = \frac{d\Pi/dw}{dCS/dw} + 1$ the equation in the proposition follows. *Q.E.D.*